1 Project activities

The primary research activities of this project during year 1 can be summarized as follows:

- We have developed a tightly coupled sediment transport/hydrodynamics model within the framework of a discontinuous Galerkin (DG) shallow water code which has been under development by the PIs and collaborators over several years. The code allows for approximation order ranging from piecewise constant (i.e., finite volume) to piecewise quadratics. In order to implement piecewise quadratics, new stability postprocessing was added to the code. The model has been implemented within the parallel Advanced Circulation (ADCIRC-DG) framework and tested on a number of applications. The results of this work have been submitted for publication to the journal *Computer Methods in Applied Mechanics and Engineering.* This work is part of the Ph.D. thesis research of Christopher Mirabito at UT Austin, who has passed his thesis proposal examination and been admitted to Ph.D. candidacy.

- The DG methodology applied to the coupled sediment transport/hydrodynamics model has been analyzed mathematically. Stability and *a priori* error estimates have been derived. This work represents the first complete error analysis for DG methods applied to the full nonlinear shallow water system.

- In collaboration with Dam Wirasaet, Seizo Tanaka and Joannes Westerink at University of Notre Dame, we have studied the performance of nodal bases on triangles and on quadrilaterals for DG solutions of hyperbolic conservation laws. These tests have shown that quadrilateral elements are more computational efficient (in terms of cost to achieve similar accuracy) over triangular elements. Based on these results, we plan to incorporate quadrilateral elements into our shallow water code in the near future.

- We have studied and begun development of a finite volume/DG based methodology for non-phase resolving wave models. This research is the work of Jessica Meixner, a graduate student at UT Austin. Ms. Meixner has spent the first year of the project learning wave theory, becoming familiar with the SWAN (Simulating Waves Nearshore) model, which is a well-known wave model developed at TU Delft, and implementing her own finite volume based wave energy model. The focus of this research is to investigate p and h adaptive non-phase resolving wave models, which have not been attempted before (current wave models are low order and/or require structured grids).

- We have begun development and implementation of a novel DG method for the three-dimensional shallow water equations. The three-dimensional shallow water equations are solved over a time-dependent domain in $\mathbb{R}^3$ using a DG method in space. A key feature of the developed DG method is the discretization of all the primary variables using discontinuous polynomial spaces of arbitrary order, including the free surface elevation. In a standard Cartesian-coordinate system, this results in elements in the surface layer having mismatched lateral faces (a staircase boundary). This difficulty is avoided in the current method by employing a so-called sigma-coordinate system in the vertical, which transforms both the free surface and bottom boundaries into coordinate surfaces. The top sigma-coordinate surface, which corresponds to the free surface, is discretized using a two-dimensional triangular mesh that is extended in the vertical direction to produce a three-dimensional mesh of one or more layers of triangular prismatic elements. The polynomial spaces over these elements are constructed using an orthogonal basis, which results in a matrix-free implementation of the method. New symmetric quadrature rules for the integration of complete polynomials over triangular prisms were also developed, which require fewer integration points than other available methods of numerical integration over triangular prisms.

- An automatic mesh generator for free surface, hydrodynamic models is also being developed, implemented, and tested. This research is the work of Colton Conroy, a graduate student at OSU. The goal of the mesh generator is to automatically produce a high-quality unstructured finite element mesh starting with only a target element size and a set of points defining the boundary and bathymetry/topography of the domain. From the geometry provided, properties such as local features, curvature of the shoreline, bathymetric/topographic gradients, and approximate flow characteristics can be extracted, which are used to determine local element sizes. Techniques incorporated include the use of the so-called signed distance function, which is used to determine critical geometric properties, the approximation of piecewise linear coastline data by smooth cubic splines, a so-called mesh function used to determine element sizes and control the size ratio of neighboring elements, and a force-displacement method which iterates to improve the element quality of the mesh. The method has been implemented in Matlab and has shown to automatically produce high quality meshes, with the correct amount of refinement where it is needed to resolve all of the geometry and flow characteristics of the domain.

2 Major presentations related to this work

- C. Dawson, *Modeling Storm Surge and Sediment Transport Using Discontinuous Galerkin Methods*, Applied Mathematics Colloquium, University of Twente, The Netherlands, February 2010.

- C. Dawson, *Discontinuous Galerkin Methods for Storm Surge and Sediment Transport*, SIAM Annual Meeting 2010, Pittsburgh, July 2010.

- E. Kubatko, *A sigma-coordinate, discontinuous Galerkin method for the three-dimensional shallow water equations*, Second International Workshop on Advances in Computational Mechanics, Yokohama, Japan, March 2010.

- E. Kubatko, *Development and implementation of a sigma-coordinate, discontinuous Galerkin method for the three-dimensional shallow water equations*, The Ninth World Congress on Computational Mechanics, Sydney, Australia, July 2010.

- C. Conroy and E. Kubatko, *An advanced automatic mesh generator for hydrodynamic models*, The Ninth World Congress on Computational Mechanics, Sydney, Australia, July 2010.

3 Educational and outreach activities

Several graduate students are involved in the project: including Christopher Mirabito and Jessica Meixner at UT Austin, both in the computational engineering, science and mathematics program, and Colton Conroy at The Ohio State University in the Department of Civil and Environmental Engineering.

In addition to training students in interdisciplinary research and high performance computing, we have performed a number of outreach activities to the larger community. Dawson's research on hydrodynamic modeling has been featured in SIAM News, International Science Grid This Week, IEEE Spectrum and ComputerWorld, as well as several local media outlets.